Short and Sweet

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A Live Experience of Four-Dimensional Structure

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Abstract

Selective attention is well known for 2D patterns and perceptual alternations are well established for 3D structures projected into 2D, such as the Necker cube. Here, these concepts are extended to the spatial fourth dimension in the form of the mathematical structure of the 4D hypercube. In orthographic projection, its 2D outline figure has multiple and highly dynamic percepts of up to 28 different 3D interpretations, which correspond to local 3D views of the 4D hypercube. Thus, the spontaneous operations of perceptual processing can provide direct insight into conceptual structure in the fourth dimension.

Keywords

3D perception, attention, depth, eye movements, perceptual organization, rivalry/bistability, spatiotemporal factors

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Attentional selectivity in two dimensions has been well known, at least since the publication of the Marroquin texture in Marr (1982, p. 50) shown in Figure 1A, but in fact going back to the chessboard illusion of one of the founders of Gestalt Psychology, Friedrich Schumann (1904). Marroquin's texture has the (accidental) property of evoking pronounced selectivity to local organization. At first glance, it is an almost uniform texture of small dots, but with scrutiny a vast array of circular structures (circles, lenticles, and rings of florets) successively announce themselves, persist for a while, and then evaporate. While some are concentric with the foveal fixation location, most are peripheral to it. They cannot be attributed simply to eye movements and contrast adaptation (as can the "monocular rivalry" effects of Atkinson et al., 1973), because those would occur for all pattern regions of the same configuration simultaneously across the image, whereas the attentional enhancement is typically

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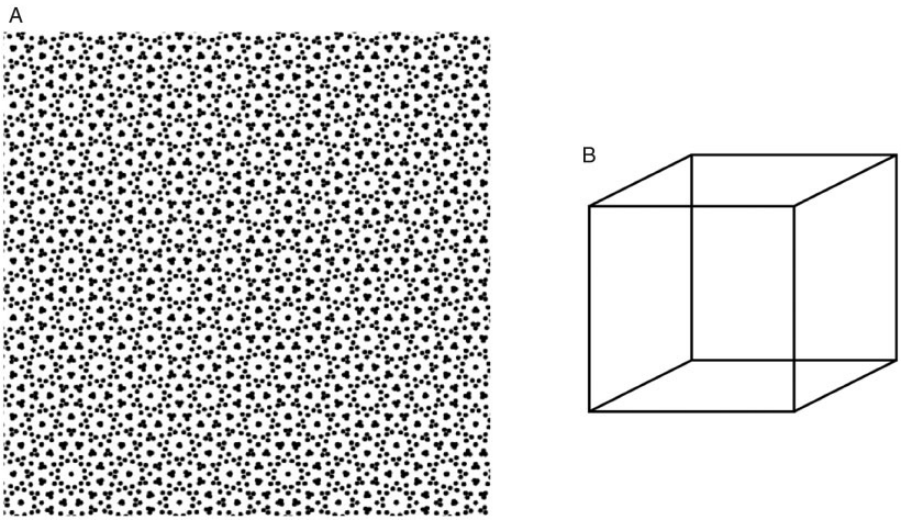

Figure 1. A: The Marroquin texture (Marroquin, 1976), in which a variety of circles and almond shapes will announce themselves and then evaporate during fixation, without conscious control and always within the flat 2D plane. B: Orthographic projection of a cube, known as the Necker cube. It is immediately seen as a 3D structure, and prolonged scrutiny will reorganize the cube as being viewed from upper left or lower right viewpoints (Necker, 1832).

to one or two local circular structures at a time (at a typical viewing distance of about 40 cm).

With close viewing, at say 10 cm, the same kinds of circles are perceptually highlighted, but in large numbers simultaneously. With as many as 40 overlapping medium-sized circles available, a dozen or so may be perceived at any given time, together with an array of smaller and some larger circles, dynamically dissolving and reforming across the image. No three-dimensional (3D) aspect of this flux of figural progressions is ever perceived, in contrast to the simple figure of the Necker cube (Figure 1B), which seems to stand out from the page in a pronounced depth impression. Here the alternation is of the viewpoint of the figure as a whole.

Polytopes in 4D (and higher dimensional) space are well-developed in contemporary mathematics. The 4D hypercube (or tesseract), for example, is shown in an orthographic (parallel) representation in Figure 2A. The edges are all the correct (unity) length, but the angles at the vertices are adjusted to allow them all to lie in the flat plane of the printed image (as was the case for the Necker cube in Figure 1B). There are an infinite number of such projections, the most symmetric of which is the one shown here. This is a highly ambiguous figure, and I will argue that, in addition to the standard ambiguities of well-known illusions, the figure demonstrates a pronounced 3D rather than 2D attentional selectivity that has perhaps not been previously reported.

Mathematical 4D structures are reportedly quite challenging to visualize as a 4D object due to our lack of familiarity with a 4D reality (Coxeter, 1948). There is, however, a direct perceptual way to experience 4D objects. Steady fixation of this figure will activate a series of percepts, as follows:

1. A planar floret-like figure of four-axis symmetry, somewhat reminiscent of overlaid large format paperclips (see Figure 3B center, with added elements to enforce flatness on initial

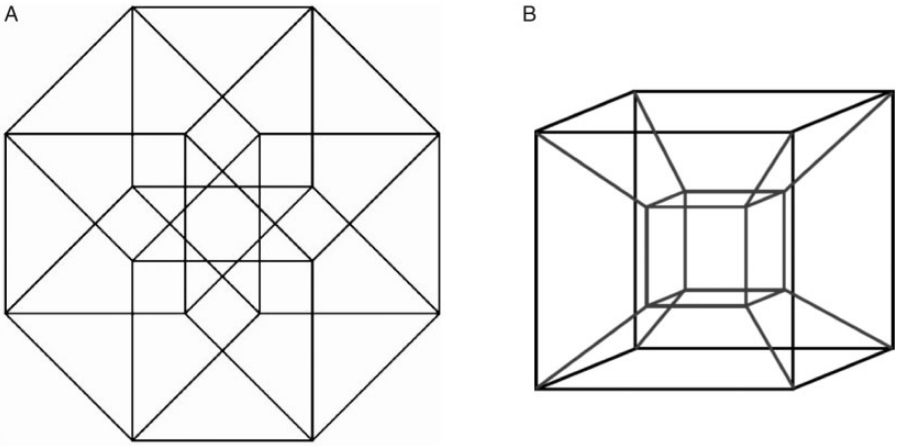

Figure 2. A: Orthographic (parallel) projection of a 4D hypercube. Although it tends to be seen initially as a kind of hexagonal hardware nut, prolonged scrutiny will successively reveal the 3D projection of each 4D cubic "face" of the mathematical hypercube. B: Perspective projection of a 4D hypercube. In this form, the figure tends to resist selective interpretations. Note that the two representations have the same connectivity of the 24 edges, with 4 per vertex, although each cube is the identical 2D shape and size in A, whereas they are scaled and distorted in B to convey the 4D equivalent of perspective described in the text.

viewing). However, the symmetry is not sufficient to hold it flat, and it soon develops into Percept 2.

2. A 3D whole-figure reminiscent of a hexagonal hardware nut though without a proper hole. This whole structure may be perceived in one of four oblique 3D slants around the horizontal or vertical axis of rotation (see Figure 3A).
3. Dominated by a single 3D Necker-cube structure, of which there are 8, with the other 7 being deemphasized or flat (see Figure 3B, outer ring). The order of salience may be random or sequential.
4. Dominated by a symmetric pair of 3D Necker-cube structures (see Figure 3C).

The typical experience of the orthographic projection of the 4D hypercube in Figure 2A is that it first appears as the 2D symmetric planar figure of option 1, then pops into one of the 3D depth structures of option 2. Most of the time, perception then follows option 3, selective attentional alternations among all possible cuboid structures contained within the figure (as itemized in Figure 3B). There are a total of eight such 4D cubic "faces" in the hypercube, but 16 different views may be obtained because each component cube can be seen from either above or below, in Necker-cube alternation. Mathematically, this perceptual experience corresponds to the 4D version of viewing each individual face of a physical 3D cube in succession (or their combinations, as in Figure 3C). This experience adds the time dimension to the 4D spatial dimensions in the 4D equivalent of the manner that an ant would experience the nature and connectivity of a 3D cube while crawling over its 2D faces (although the 3D perceptual sequence requires a cognitive interpretation to be understood as a 4D trajectory). Note that the temporal order of the appearances of the cubic "faces" tends to be random with free eye movements, but that the order can be controlled by focusing on a particular 2D face of the currently salient cube, which will then encourage transition to the particular cube adjacent to that face. Just as we can attend to each face of a (transparent) 3D cube such as the one in Figure 1B, the perceptual system has the capability of attentionally

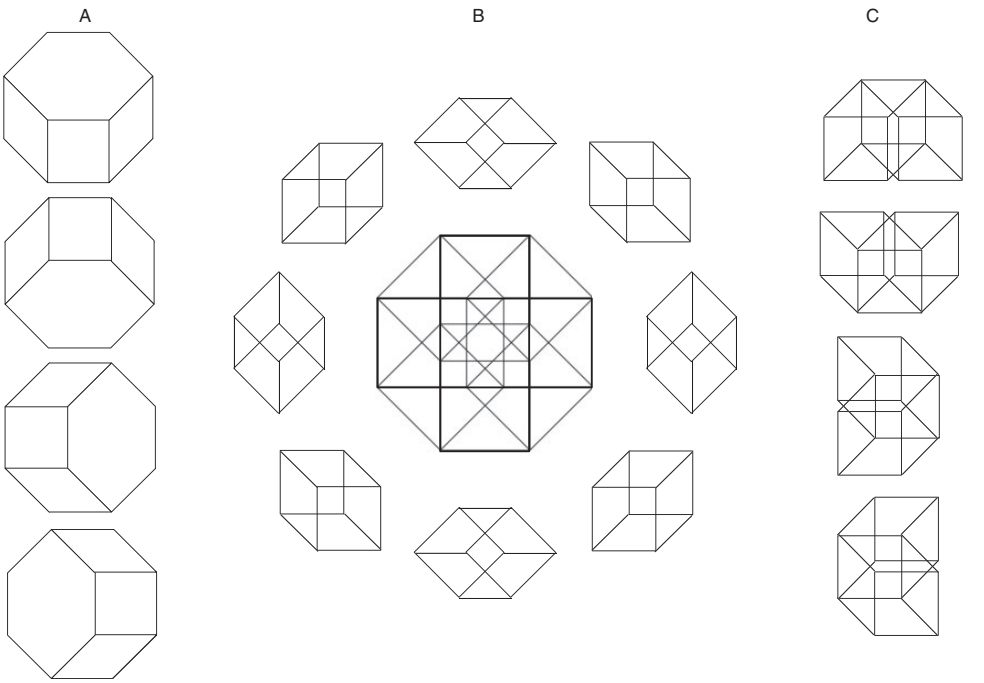

Figure 3. Illustrations of one 2D (center) and 28 3D percepts of Figure 2A (including alternate viewpoints in the left a right columns, A & C).

enhancing each cubic "face" of the 4D hypercube in turn, particularly when it is projected in the orthographic (parallel) projection of Figure 2A. In this way, we can experience the nature and connectivity of the hypercube as a 4D object.

It is important to distinguish between the Necker-cube alternations in Figure 1B and the variety of hypercube alternations in Figure 2A. While Necker-cube alternations are different views of the whole 3D object made up of the same lines forming the same cube, the 4D alternation of the orthographic projection of Figure 2A takes the form of a selective type of attention to the 3D interpretation of only one part of the image at a time, with the rest appearing flat (cf. Tyler & Kontsevich, 1995). This expands the concept of multiple selective attention from the 2D case of Marroquin to the realm of the selective perception of 3D structure.

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