Invariance of the slope of the psychometric function with spatial summation

Melanie J. Mayer

Program in Experimental Psychology, University of California, Santa Cruz, California 95064

Christopher W. Tyler

Smith–Kettlewell Institute of Visual Sciences, 2232 Webster, San Francisco, California 94115

Received November 19, 1984; accepted March 27, 1986

The high-threshold probability summation model for improved detection of near-threshold gratings with increased spatial extent of the patterns assumes that the detection psychometric function is a Weibull function. This model predicts that the slope of the psychometric function should not change as the number of mechanisms stimulated increases, although other models predict that the slope should vary. We confirm for a two-alternative, forced-choice paradigm that the slope parameter does not vary systematically with spatial frequency or with number of periods of the grating, although there are reliable differences in average slope between observers.

When detecting near-threshold visual patterns, humans can use stimulus redundancy to improve performance. Thresholds are lower when the pattern is repeated across either time or space.[1–3] One account of this empirical result is that pattern extension creates the opportunity for multiple observations that are combined to improve detection.[4] The most common model of the processes involved assumes that each of several observations leads to one of two sensory states (detect or not), and that these states are then combined by a logical OR to make an overall decision: "Say 'yes' when there is at least one detection across all observations or mechanisms." This is referred to as probability summation. The simplest version of this model, high-threshold probability summation, further assumes that a detect state can occur only when a pattern is actually present; there are no sensory false alarms. In a yes–no detection task then, a "yes" response occurs only when there is a detect state or as a result of guessing, which is independent of the sensory states. (See, e.g., Sachs *et al.*,[5] Quick,[6] and Nachmias.[7])

One outcome of high-threshold probability summation is that the rate of improvement in sensitivity with the number of observations is an inverse function of the slope of the psychometric function. Specifically, if the probability that a mechanism will detect a stimulus is described by a Weibull[8] function, then

$$\alpha = (\Sigma\alpha_i^{-\beta})^{-1/\beta}, \tag{1}$$

where α is the overall threshold contrast, α_i is the threshold contrast of individual mechanisms, and β is the slope parameter, assuming all slopes are equal.[9] Then if all mechanisms are equally sensitive (all α_i are equal), at threshold

$$S = 1/\alpha \propto N^{1/\beta}, \tag{2}$$

where S is the overall, combined sensitivity, and N is the number of mechanisms or observations.[3]

A key prediction of high-threshold theory is that β, the slope parameter, should be constant independent of the stimulus and independent of the number, N, of observations or mechanisms. Most researchers have assumed that this is the case. However, Nachmias[7] has shown that this assumption fails when comparing β for bipartite fields versus 12 cycles/degree (c/deg) gratings and for yes–no versus forced-choice paradigms. He also shows with a rating experiment that the slope parameter decreases as more lenient ratings for a "yes" response are used.

Furthermore, Wilson and Bergen[10] suggest that β may increase with the spatial frequency of the gratings to be detected and with the number of observations/mechanisms. Their suggestion is based on the assumption that the psychometric function for an individual mechanism is best described by a log cumulative normal as opposed to a Weibull function. For yes–no data, when high-threshold probability summation is used on the log normal functions but the results are fitted by a Weibull function, β increases with number of mechanisms. They also assume that increasing spatial frequency in a fixed viewing area increases the number of mechanisms stimulated. However, they did not extensively test either of these ideas.

The experiments reported here were designed to examine whether β varies with the number of mechanisms stimulated when the guessing rate is controlled by two-interval, forced-choice procedures. We tested this using Wilson and Bergen's[10] assumption that increasing spatial frequency increases the number of mechanisms. However, since this confounds potential changes in β with spatial frequency rather than number of mechanisms, we also tested whether β varied with the number of periods of a grating of fixed spatial frequency.

METHODS

Observers

Two women (MM, DD) and two men (JB, JP) were observers in the experiments. MM's myopia and minor astigmatism

0740-3232/86/081166-07$02.00

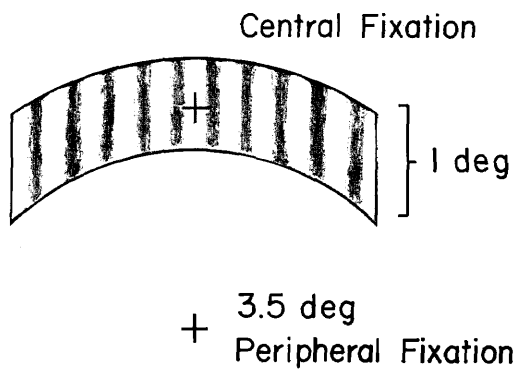

Fig. 1. Appearance of the 4-deg, horizontal, annular-segment window through which sinusoidal gratings were viewed. Fixation was either foveal or so as to place the window at a constant eccentricity of 3.5 deg in the periphery. Spatial frequency within the 4-deg-wide window was varied for Experiment 1. In Experiment 2, the number of periods of a 12-c/deg grating was varied by adjusting the width of the window symmetrically about the upper vertical meridian.

were corrected with glasses. DD and JB had slight amounts of astigmatism (≤ 1 D), and JP was emmetropic. MM was a highly practiced psychophysical observer; the others were experimentally naive.

Apparatus

A small computer (Apple II+) with custom interface generated and controlled Michelson[11] ($L_{\max} - L_{\min}/L_{\max} + L_{\min}$) contrast, spatial frequency, and timing of vertical sinusoidal gratings. They were displayed on a monitor with P-31 phosphor (Hewlett-Packard 1332A) at a space-average luminance of 40 cd/m^2. A cardboard surround, matched in color and average luminance to the gratings, extended 2.6–7.5 deg visual angle around the display. Gratings appeared in a horizontal annular-segment window, 1.0 deg high and 0.33–4.0 deg wide, whose smaller radius was 3 deg (Fig. 1). The window was always symmetric about the vertical meridian. Observers sat with their heads resting comfortably on a chin rest, viewing the display binocularly and with natural pupils at a distance of 152.4 cm.

Procedure

Detection trials were presented as two-interval, forced-choice (2IFC) with each interval 500 msec long and separated by 200 msec. Onset of each observation interval was cued by a brief computer-generated tone. Each field size and spatial frequency was run in a block of 300–320 trials, with order counterbalanced across replications.

After preliminary trials in which contrast was adjusted for approximately the 75% correct response level, the method of constant stimuli was used to generate psychometric functions. Usually 5 contrast levels at 0.12-log-unit steps were presented either in random or in blocked, counterbalanced ascending and descending order, with 60 trials per level. On replications where psychometric functions were steep, as they were for two observers, the number of contrast levels was reduced to 4 and the number of trials per contrast increased to 80 to improve statistical reliability of parameter estimates.[12]

Gratings appeared in the first or second observation intervals in approximately equal numbers and in random order. After viewing a trial, the observer entered "1" or "2" on the computer keyboard to indicate in which interval the grating appeared. A third button was also available to reject trials in which attention or fixation wavered. Observers were instructed to maintain fixation on marks in the center of or 3.5 deg from the gratings and to reject trials on which they were unable to do so. Visual monitoring of the observers during practice tests verified that they followed instructions, and their reduced spatial frequency resolution in the periphery (see Results) confirmed that accurate fixation was maintained. Incorrect responses were followed by a low, buzzing tone from the computer. Individual trials took about 4 sec, and data for each psychometric function required 30–40 min of testing, with breaks when needed.

Each observer had at least one session (1.5–2 h) of practice with all conditions before beginning each experiment. In Experiment 1, we measured the effects on threshold (α) and on slope (β) parameters of changing number of mechanisms by varying spatial frequency from 2 to 36 c/deg. The full 4-deg-wide annulus was viewed either with central fixation or with its lower edge 3 deg from fixation and centered on the vertical meridian in the upper visual field. The former condition approximates the central viewing used in Wilson and Bergen's[10] experiment, while the latter condition places the annulus in an area of approximately homogeneous sensitivity for any one spatial frequency.[3] In Experiment 2, we measured the effects on α and β of changing number of mechanisms by varying stimulus area. Psychometric functions were derived for detecting 4–48 periods of a 12-c/deg grating viewed peripherally. For DD and MM we directly assessed homogeneity of the peripheral test area by measuring sensitivity across the annulus in four-cycle segments.

Data Analysis and Predictions

All 2IFC psychometric functions were fitted by a Weibull[8] function

$$P(c) = 1 - (1 - 0.5)\exp[-(c/\alpha)]^{\beta}, \tag{3}$$

by using the STEPIT algorithm[13] to generate maximum-likelihood estimates[2] of α and β, the threshold and slope parameters, respectively. Here, $P(c)$ is the probability of correct response for contrast c, and 0.5 is the guessing parameter for 2IFC.

Wilson and Bergen[10] predicted increasing β by probability summing a log cumulative normal (LCN) psychometric function, $P_1(c)$, according to the high-threshold model, using

$$P_N(c) = 1.0 - [1.0 - P_1(c)]^N, \tag{4}$$

where N is the number of mechanisms and $P_N(c)$ is the probability of correct response at contrast c for N mechanisms [compare Eq. (A9) of Wilson and Bergen[10]].

The model in Eq. (4) must be modified somewhat to accommodate 2IFC data, where the guessing rate, γ, is forced to remain at 0.5. The modification consists of transforming the 2IFC data to a hypothetical function with guessing removed, fitting them to an LCN function, and then, following the procedure suggested by Wilson and Bergen, probability summing the transformed LCN function. Finally we determine the change in β obtained when the summed functions are fitted by Weibull functions. Specifically, we first used each observer's average α and β for the lowest number of mechanisms condition (either lowest spatial frequency or smallest number of periods) to generate a nine-level Weibull psychometric function. Then we transformed this function according to the usual correction for guessing,

$$\Psi(c) = [P(c) - \gamma]/[1 - \gamma], \quad (5)$$

where $\Psi(c)$ is the hypothetical psychometric function with guessing removed and $P(c)$ is the 2IFC psychometric function. The transformed points were plotted on log contrast by probability graph paper and fitted with a straight line to give a log cumulative normal (LCN) curve whose α and β matched those of the original function when fit with the Weibull fuction by STEPIT as described above. Then, assuming high-threshold probability summation, this best-fit LCN curve was used to calculate psychometric functions, $\Psi_N(c)$, for $N = 1$–48 using

$$\Psi_N(c) = 1.0 - [1.0 - \Psi_1(c)]^N, \quad (4')$$

where $\Psi_1(c)$ is the LCN psychometric function for $N = 1$ and with guessing removed. Each resulting curve was then fitted with the maximum-likelihood STEPIT Weibull function to estimate α and β.

If the actual data conform to the LCN psychometric function, then β should increase with increasing number of mechanisms stimulated when the functions are fitted by Weibull functions. Conversely, if the Weibull function is an accurate reflection of the form of the psychometric function, probability summation should produce no change in β. When comparing predicted with measured parameters, we assume that N is the number of periods viewed of a 12-c/deg grating when grating size varies or that N is the number of periods in 1-deg visual angle when spatial frequency varies (i.e., equal to the spatial frequency number).

RESULTS

Experiment 1: Spatial Frequency

Slope parameter β was approximately constant for changing spatial frequency in the fovea and also at 3.5 deg in the upper visual field (Figs. 2 and 3, respectively). High-threshold, constant β predictions (dotted lines) were calculated from mean β across spatial frequencies for each observer. Predictions for increasing β assuming LCN rather than Weibull psychometric functions (dashed lines) were also calculated for each observer, as described in the Methods section. Note that differences between constant β and LCN predictions become larger for higher values of β.

There is considerable variation in the slope parameter *among observers*, with averages of 3.24 (± 0.39) for DD, 5.08 (± 0.72) and 4.35 (± 0.54) for MM, and 2.45 (± 0.24) for JB.

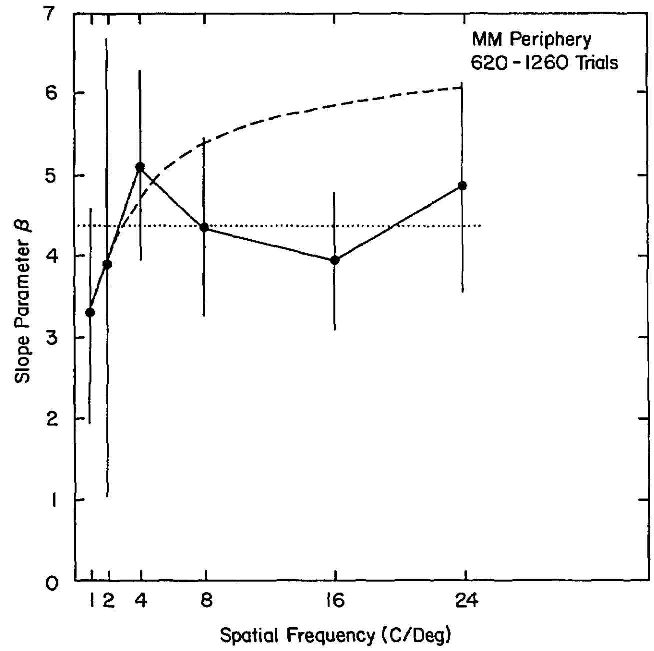

Fig. 3. Slope parameter β as a function of spatial frequency of sinusoidal gratings 3.5 deg in upper visual field for observer MM. Graphing conventions are the same as those in Fig. 2.

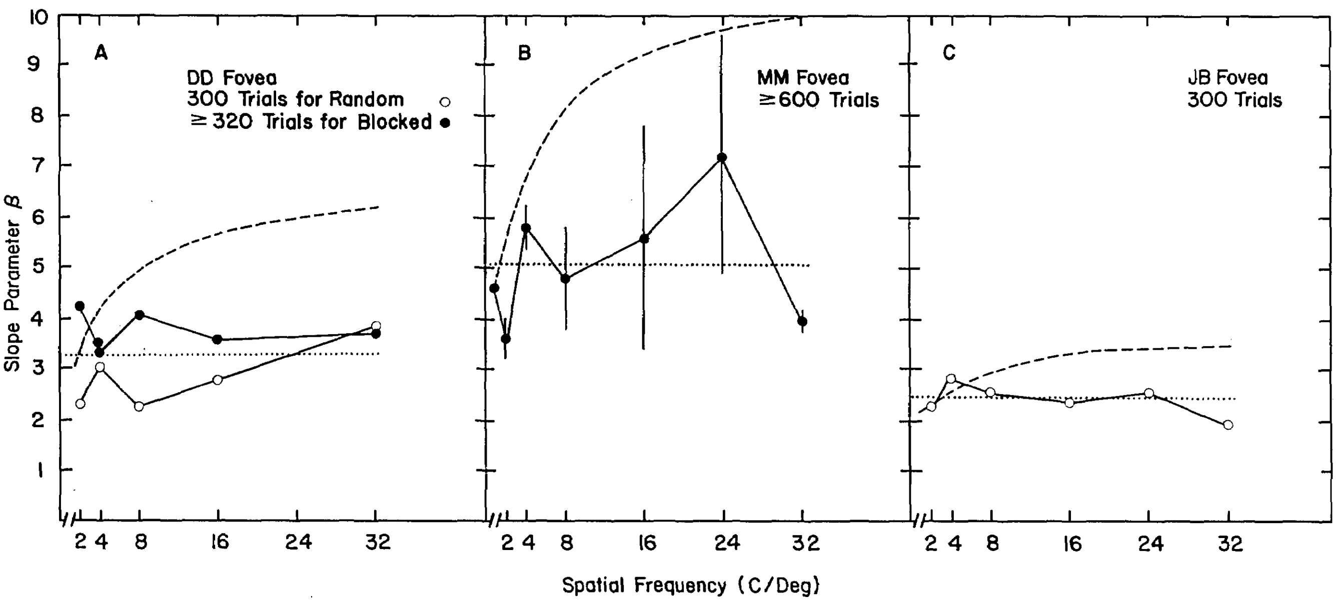

Fig. 2. Slope parameter β as a function of spatial frequency of sinusoidal gratings viewed foveally by three observers. Data are plotted for blocked contrast with filled circles and for random contrast with open circles. Dotted line is the high-threshold prediction of constant β; dashed line shows predicted increase in β assuming log cumulative normal rather than Weibull psychometric functions. Numbers of trials is per data point. Data points for all sessions are shown for DD and JB. For MM, vertical bars indicate mean 95% confidence interval (±1.96 SEM).

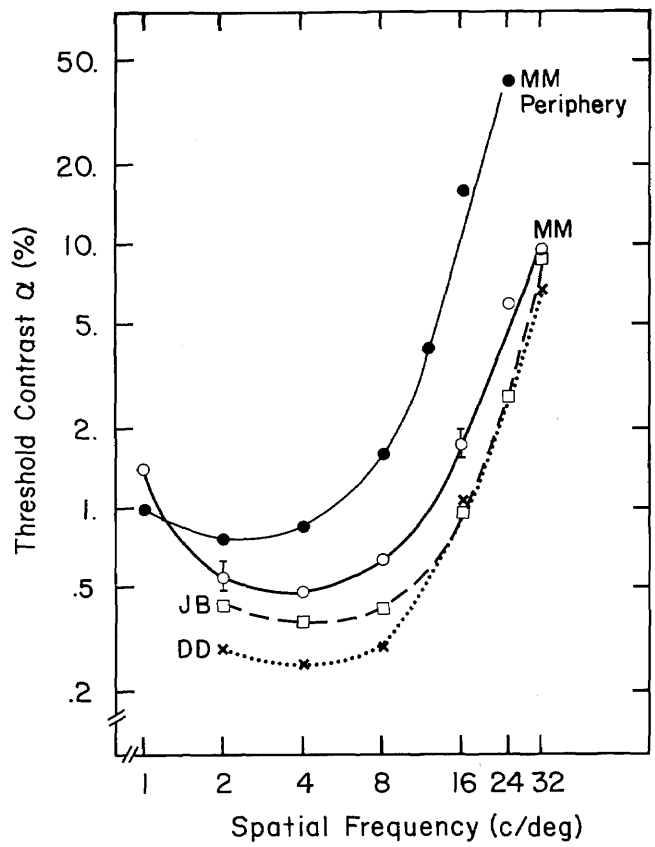

Fig. 4. Threshold contrast α as a function of spatial frequency. Circles show MM's data in fovea (O) and periphery (●). Foveal curves for observers DD (X) and JB (□) have been shifted down 0.15 log unit for clarity. The 95% confidence intervals are shown with vertical bars or are within the symbol size.

(The 95% confidence intervals in parentheses are ±1.96 standard error of the mean (SEM), assuming normal distributions.) Higher slopes for MM may be because this observer's psychometric functions had the various contrasts run in blocks of trials. Though blocked versus random contrast made no difference for DD, MM's β's with contrast randomly presented are generally lower, e.g., 2.7 and 3.12 for other conditions not reported here.

Observers DD and JB have little if any change in slope parameter with spatial frequency. MM's data, while having higher β's, are also more variable. However, 95% confidence intervals for all points beyond the first of the curve in Fig. 2(b) are outside the LCN predictions. Also, root-mean-square deviations (RMSD's) comparing MM's data with predictions show better fits for constant β in both fovea (Fig. 2) and periphery (Fig. 3). [RMSD(df = 6) = 3.03 for constant versus 8.47 for LCN in fovea; RMSD(df = 5) = 1.55 for constant versus 2.50 for LCN in periphery.]

Figure 4 shows that threshold contrast α has the expected[14] U-shaped variation with spatial frequency. Foveal sensitivity is best at about 4 c/deg for all observers (open symbols and X's). Peripheral thresholds for MM (filled circles) are generally higher than foveal at all spatial frequencies except 1 c/deg, and maximum sensitivity is at approximately 2 c/deg. These results validate that peripheral fixation was maintained, since the spatial frequency tuning shows the shift to lower spatial frequencies by a factor of 2 expected at 3 deg eccentricity.[15] They also highlight the problem of looking for probability summation effects by varying spatial frequency. If probability summation were operating, sensitivity should be increasing with increased spatial frequency; however, beyond 4 c/deg in the fovea and 2 c/deg in the near periphery, sensitivity is decreasing. If there is probability summation with increasing spatial frequency, it is confounded with other sensitivity changes.

Experiment 2: Number of Periods

To study the effect on the psychometric function of probability summation over number of periods, we presented the stimuli at 3° in the periphery to stimulate areas of approximately equal sensitivity as the number of periods was increased. Figure 5 shows that, when periods of a 12-c/deg grating in the periphery were varied, the low values of measured β made quantitative predictions for LCN difficult to distinguish from those of constant β. Obtained curves do not differ greatly from either prediction. RMSD's favor constant β for MM (1.59 versus 3.55), but are about equal for the other observers (DD, 1.27 versus 1.20; JP, 1.26 versus 1.20 for constant versus LCN, respectively). However, once again there are significant differences in mean slope among observers: 2.89 (± 0.35) for DD, 4.3 (± 0.46) for MM, and 2.28 (± 0.38) for JP.

Figure 6 shows the results for testing homogeneity of sensitivity across the annular display area. Threshold contrast α is relatively constant for four-period segments across the annulus for DD and MM, although, interestingly, both have slightly higher thresholds at the segment centered on the vertical meridian.

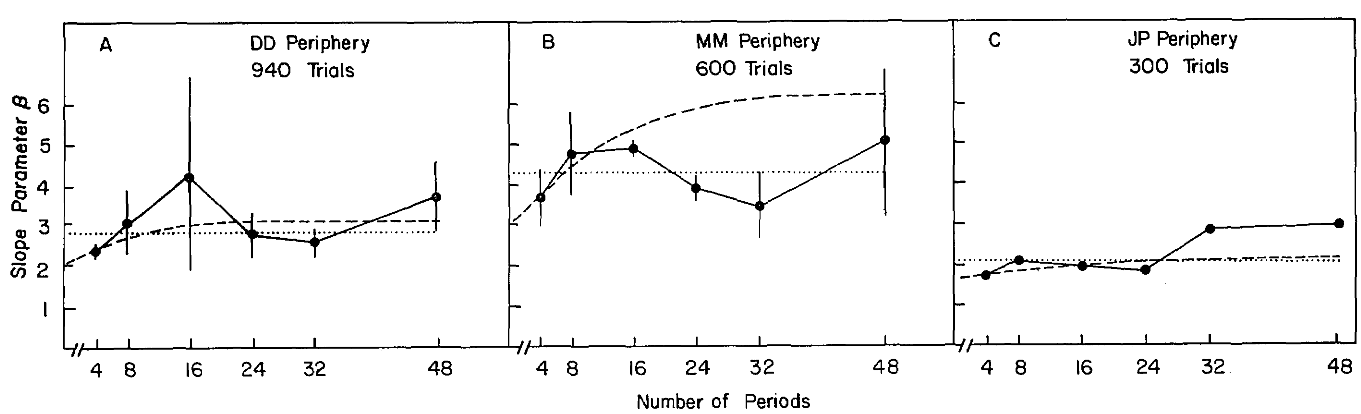

Fig. 5. Slope parameter β as a function of number of periods of a 12-c/deg grating for three observers with 3.5-deg peripheral viewing. Graphing conventions are those in Fig. 2.

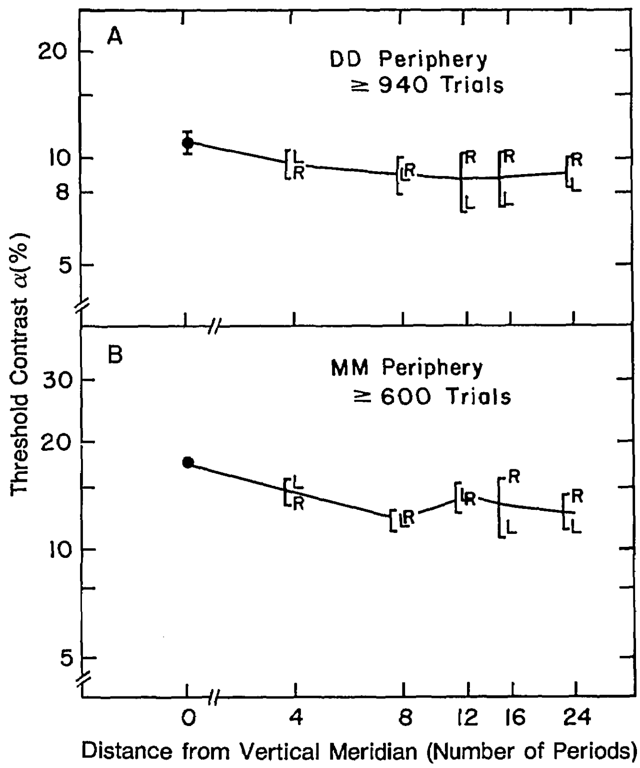

Fig. 6. Threshold contrast α for two observers viewing four-period segments of 12-c/deg grating at 3.5 deg in upper visual field. The segments appear within the annular window and are at varying distances from the vertical meridian. L's show positions in the left visual field, R's in the right. Confidence intervals are 95%.

Figure 7 shows the decrease in detection threshold with increased number of periods of the grating display. For all three observers, threshold decrease in the initial segment of the curve is steeper than that predicted by the high-threshold theory (dotted lines and X's) and approaches a slope of −1.0 (−0.87 for DD, −1.00 for MM, −1.06 for JP between 4 and 8 periods; −0.87 for MM, −1.10 for JP between 4 and 16 periods). Slopes for the last four points in each curve are shallower, especially for MM and JP. (Data for DD and MM have been normalized to eliminate practice effects between testing sessions. The data for each complete session was divided by the mean threshold for that session and then multiplied by the mean threshold over all sessions for each observer. This has the effect of eliminating between-session variability from the error term, while leaving the mean threshold values and the shape of the function unaffected. High-threshold theory does not address practice effects, but since thresholds tested later in the experiment were generally lower, normalized thresholds provide a more accurate test of theoretical predictions based on intrasession variability. Error bars show the 95% confidence interval.)

DISCUSSION

Two methods were used to manipulate the number of mechanisms contributing to detection of sinusoidal gratings: (1) varying the spatial frequency of a grating appearing in a fixed display size, which changes the number of periods visible as well; and (2) varying the number of periods, and hence the area, of a grating of fixed spatial frequency.

Slope Parameter

Results from both manipulations give very little reason to reject the assumption of constant slope parameter β for a particular observer under fixed testing conditions (Figs. 2, 3, and 5). There is no evidence of systematic decrease or increase in β with number of mechanisms.

The slope parameter does appear to vary *among observers*

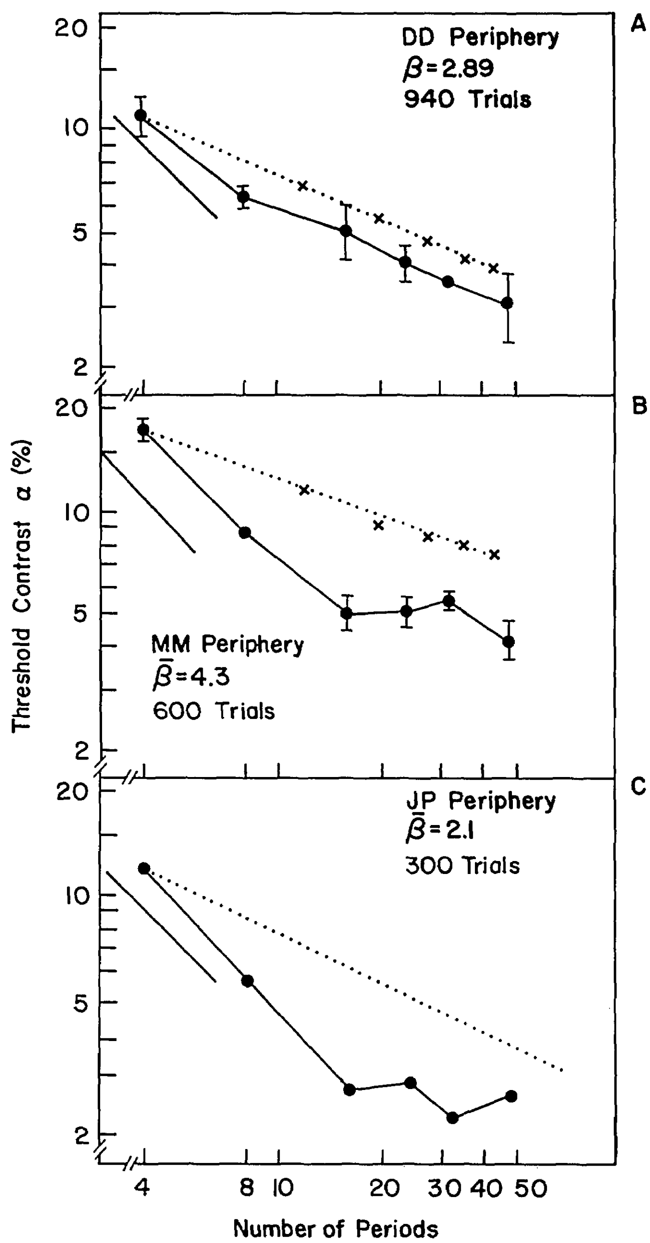

Fig. 7. Threshold contrast α for three observers as a function of size of 12-c/deg grating viewed at 3.5 deg in upper visual field. Data are plotted with filled circles and connected by solid lines. Solid line to left shows slope of −1 (full summation). For DD and MM, X's connected by dotted lines show predictions for combining appropriate annular segments according to high-threshold probability summation [Eq. (1)]. For JP, the dotted line shows high-threshold predictions assuming homogeneous sensitivity across the annulus. Confidence intervals (some within size of symbol) are 95%.

(Figs. 2 and 5). This agrees with Table 1 in Nachmias,[7] and observer differences are taken into account in all our predictions using β. Testing conditions may play a part in this variability since some psychometric functions were generated with blocked rather than random contrast variation. Although these test conditions seem to have little or no effect on DD's slope parameters (Fig. 2A), MM's slope parameters appear shallower for random contrast psychometric functions than for blocked contrast. This agrees with Nachmias's observation[7] that β may change with method of data collection.

The increase in β with *spatial frequency* between 0.5 and 16 c/deg noted by Table 3 of Wilson and Bergen[10] is not found here. Conditions shown in Fig. 2 compare best with Wilson and Bergen's measurements for frequency of seeing foveally viewed cosinusoidal gratings. All observers' data in Fig. 2 clearly contradict the predictions from assuming an LCN rather than a Weibull psychometric function. Differences between Wilson and Bergen's and our data might be explained by differences in testing procedures. They used yes–no testing in which the guessing parameter γ can vary, while we used 2IFC, where γ is fixed at 0.5. In a high-threshold framework even with correction for guessing, Nachmias[7] has shown that β decreases as γ increases. The data in Wilson and Bergen's Table 3 are not corrected for guessing or analyzed for trends in γ with spatial frequency, but guessing rates were generally less than 10%.[16] The 1.33-fold increase in β for T stimulation shown in their Table 3 is within the range that Nachmias[7] found resulting from changes in γ, however; therefore, their increase in β may be associated with decreased γ with higher spatial frequency. We note also that their slope parameters do not consistently increase. Between 8 and 16 c/deg, the highest frequency that they tested, β's *decreased*.[17]

Averaging observers' data also gives no symmetric increase in β with spatial frequency (3.05, 3.98, 3.51, 3.72, 4.9, and 3.23 for 2, 4, 8, 16, 24, and 32 c/deg, respectively). As Fig. 3 shows, testing at 3.5 deg in the more homogeneous upper visual field reveals no systematic variation of β with spatial frequency either.

Our assumption that N in Eq. (4′) is the number of periods in a 1-deg-wide field may be in error by some factor of the number of cycles optimally stimulating the units of integration for probability summation. However, because the data have no systematic trend to increase, multiplying N by some scaling factor will not change the poor fits between data and LCN predictions.

To avoid confounding the number of mechanisms with spatial frequency and its associated sensitivity variation, our subsequent experiments manipulated the number of mechanisms by varying the *number of periods* of a fixed spatial frequency grating. We tested at 3.5 deg from fixation in the upper visual field to avoid confounding sensitivity measurements by effects of retinal inhomogeneity as well. The relative flatness of curves in Fig. 6 shows that our objective was accomplished and agrees with the results of Robson and Graham[3] (their Fig. 2) for similar measurements. The slight increase in thresholds for both observers on the vertical meridian suggests radial asymmetry of sensitivity even at 3.5 deg from the fovea and is consistent with other (unpublished) data from our laboratory.

The slope parameter does not vary systematically with the number of periods; however, differentiating between constant and LCN β proves more difficult since curves for the two predictions are quite close (Fig. 5). For MM, whose average β is high enough to give relatively different predictions at larger N, data are closer to constant β. (Note that predictions at lower N are constrained by the fitting procedures described in the Methods section.)

The fact that average threshold α systematically decreases by a factor of 4–5 (Fig. 7) shows that increasing the number of periods does produce a substantial amount of summation. The fact that β nevertheless remains relatively stable implies that the Weibull function is a good approximation to the true shape of the psychometric function for individual mechanisms.

Sensitivity

Although a number of experiments have studied probability summation psychophysically by increasing the area of grating patterns,[1,3,18–23] only Robson and Graham[3] use peripheral gratings and luminance-matched surround, as we did. In the periphery, gratings can be presented to approximately equally sensitive adjacent retinal areas (Fig. 6).

Like Robson and Graham,[3] we find that increasing the number of periods (area) of 12-c/deg vertical grating improves its detectability at least out to 48 periods (1 × 4 deg), as shown in Fig. 7. However, our data suggest that there may be two summation rules operating. Complete (slope = −1.0) or nearly complete summation may extend to between 6.0 and 11.4 periods, depending upon the observer. Beyond the complete summation region, sensitivity appears to improve more slowly with the number of mechanisms, with a slope approximating the $-1/\beta$ predicted by the high-threshold probability summation model [Eq. (2)]. The two-segment form is most apparent in curves for MM and JP (Figs. 7B and 7C), but even for DD (Fig. 7A) the 4–8-period segment is considerably steeper than predicted by the high-threshold model. (For predictions in Figs. 7A and 7B, each observer's slight variation of sensitivity for individual segments (Fig. 6) has been taken into account.) Because the region of complete summation is more extended than previously reported,[3] we plan further studies of the lower end of the curve to determine the factor(s) that differentiate our results from previous work.

CONCLUSION

The slope parameter of psychometric functions showed no systematic variation with procedures intended to manipulate the number of mechanisms activated (N). This implies that the Weibull function is a good approximation to the true shape of the psychometric function for individual mechanisms, since only in this case will the β parameter remain constant over summation.[24] The invariance with N held within both methods is used to measure the psychometric functions, either blocked or randomly interleaved contrast levels. However, for one observer the slope parameter was steeper for blocked than for randomly interleaved contrasts. This finding is contrary to the high-threshold probability summation model, which does not predict uncertainty effects.[25] There were also consistent differences in the slope parameter between observers within methods. Both the individual and methodological differences suggest that β should be measured in experiments that use the parameter to test predictions about different models of summation performance.

ACKNOWLEDGMENTS

We thank Joseph Hary for programming assistance, Deborah Dalke for laboratory work, and Norma Graham for discussions of interpretation of the differences between our experiments. This research was supported by National Institutes of Health grants EY 2445 and RCDA EY 247 to M. J. Mayer, and EY 3884 to C. W. Tyler, and by the Smith-Kettlewell Eye Research Foundation. A preliminary report of these results was presented at the annual Association for Research in Vision and Ophthalmology meeting, Sarasota, Florida, 1984.

REFERENCES AND NOTES

1. E. R. Howell and R. F. Hess, "The functional area for summation to threshold for sinusoidal gratings," Vision Res. **18**, 369–374 (1978).
2. A. B. Watson, "Probability summation over time," Vision Res. **19**, 515–522 (1979).
3. J. G. Robson and N. Graham, "Probability summation and regional variation in contrast sensitivity across the visual field," Vision Res. **21**, 409–418 (1981).
4. D. M. Green and J. A. Swets, *Signal Detection Theory and Psychophysics* (Wiley, New York, 1966).
5. M. B. Sachs, J. Nachmias, and J. G. Robson, "Spatial frequency channels in human vision," J. Opt. Soc. Am. **61**, 1176–1186 (1971).
6. R. F. Quick, "A vector magnitude model of contrast detection," Kybernetic **16**, 65–67 (1974).
7. J. Nachmias, "On the psychometric function for contrast detection," Vision Res. **21**, 215–223 (1981).
8. W. Weibull, "A statistical distribution function of wide applicability," J. Appl. Mech. **18**, 292–297 (1951).
9. N. Graham, "Visual detection of aperiodic spatial stimuli by probability summation among narrowband channels," Vision Res. **17**, 637–652 (1977).
10. H. R. Wilson and J. Bergen, "A four-mechanism model for threshold spatial vision," Vision Res. **19**, 19–32 (1979).
11. A. A. Michelson, *Studies in Optics* (U. Chicago Press, Chicago, Ill., 1927).
12. S. P. McKee, D. Y. Teller, and S. Klein, "Statistical properties of forced-choice psychometric functions," Percept. Psychophys. **37**, 286–298 (1985).
13. J. P. Chandler, "STEPIT," Quantum Chemistry Program Exchange, Department of Chemistry, Indiana University, Bloomington, Indiana (1965).
14. F. W. Campbell and D. G. Green, "Optical and retinal factors affecting visual resolution," J. Physiol. **181**, 576–593 (1965).
15. J. Rovamo, V. Virsu, and R. Nasanen, "Cortical magnification factor predicts the photopic contrast sensitivity of peripheral vision," Nature **271**, 54–56 (1978).
16. H. R. Wilson, Department of Physiology and Committee on Neurobiology, University of Chicago, 920 East 58th Street, Chicago, Ill. 60637 (personal communication, 1986).
17. Size, average luminance, and temporal characteristics of the test patterns also differed between our and Wilson and Bergen's[10] experiments. For the present experiments we had 1.0 deg × 4.0 deg display area, 40 cd/m^2 luminance, and a positive one-half cycle of a 1.0-Hz square wave in each interval of a 2IFC procedure. Wilson and Bergen report 1.0 × 8.0 deg display area, 17.5 cd/m^2 luminance,[16] and Gaussian envelope with 0.25-sec time constant or one cycle of a 8.0-Hz square-wave contrast reversal in yes–no procedure. Nevertheless, none of these variables seems to be critical to Wilson and Bergen's arguments.
18. J. Hoekstra, D. P. A. van der Groot, G. van den Brink, and F. A. Bilsen, "The influence of the number of cycles upon the visual contrast threshold for spatial sine-wave patterns," Vision Res. **14**, 365–368 (1974).
19. R. L. Savoy and J. J. McCann, "Visibility of low-spatial-frequency sine-wave targets: dependence on number of cycles," J. Opt. Soc. Am. **65**, 343–350 (1975).
20. J. G. Robson, in H. Mostafavi and D. J. Sakrison, "Structures and properties of a single channel in the human visual system," Vision Res. **16**, 957–968 (1976).
21. O. Estevez and C. R. Cavonius, "Low-frequency attenuation in the detection of gratings: sorting out the artefacts," Vision Res. **16**, 497–500 (1976).
22. J. S. Tootle and M. A. Berkley, "Contrast sensitivity for vertically and obliquely oriented gratings as a function of grating area," Vision Res. **23**, 907–910 (1983).
23. J. H. T. Jamar, L. F. T. Kwakman, and J. J. Koenderink, "The sensitivity of the peripheral visual system to amplitude-modulation and frequency-modulation of sine-wave patterns," Vision Res. **24**, 243–250 (1984).
24. D. M. Green and R. D. Luce, "Parallel psychometric functions from a set of independent detectors," Psychol. Rev. **82**, 483–486 (1975).
25. N. Graham, P. Kramer, and D. Yager, "Explaining uncertainty effects and probability summation," Invest. Ophthal. Visual Sci. Suppl. **24**, 186 (1983).