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Editorial essay

Straightness and the sphere of vision

The nature of perceived visual space is an issue that has been as contentious within its field as has the nature of physical space within the field of physics. In one of the earliest systemizations of visual space, Euclid (c. 300 BC) has been interpreted as conceptualizing visual projective space in terms of a sphere surrounding the viewer. He represents a tradition that "considered our sphere of vision quite literally as a 'sphere'—an assumption, incidentally, which more nearly agrees with physiological and psychological reality than that which underlies Brunelleschi's rectilinear construction." (Panofsky 1960).

The same construct was subsequently affirmed by Leonardo da Vinci in the late 1400s and by others (see Aguilonius 1613; Abraham Bosse 1665; John Herschel 1833, 1849; Hermann von Helmholtz 1910/1962; White 1957), in terms of the 'natural perspective' of the free-viewing observer (as opposed to the 'artificial perspective' of the projection onto a flat picture plane).

The contrast with artificial perspective is illustrated in figure 1 by Bosse's illustration of the relation between the circular projection of the visual cone and the flat plane of the picture surface (inscribed with the phrase "To prove that one can neither define nor paint as the eye sees"). As a consequence, there has been an incessant debate since at least the time of Leonardo about whether straight lines in the world are perceived as straight or curved in a form of barrel distortion, and whether they should be

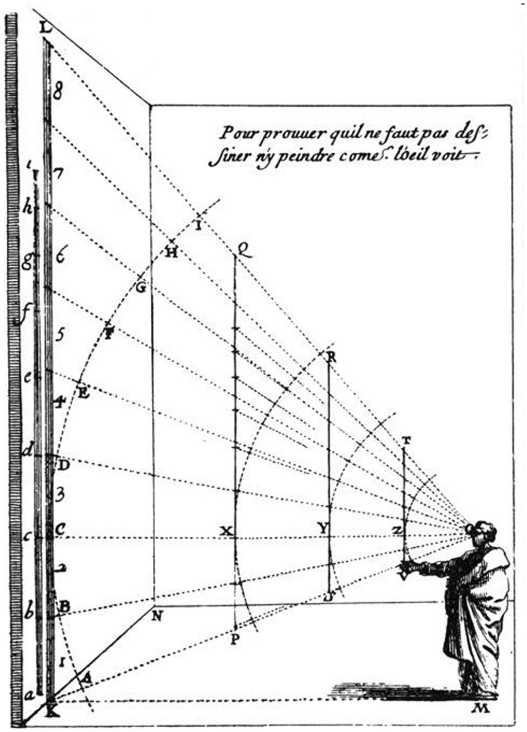

Figure 1. Diagram of the visual cone by Abraham Bosse (1665).

depicted as straight in the picture plane of the artist's canvas. Both Panofsky (1924) and Gombrich (1972), for example, argued that straight lines in natural perspective are ultimately curved, and Herschel (1849), Escher (1947—see Ernst 1976), and White (1957) developed formal schemes for the definition of curved perspective, approximating the projective geometry of the extreme wide-angle or 'fish-eye' lens. Later authors (Pirenne 1970; Hansen 1973; Anstis 1998; Liu and Schor 1998; Oomes et al 2009; Rogers and Rogers 2009) have also discussed the issue of the apparent curvature of physically straight lines.

A clear statement of the natural perspective concept may be found in Herschel's (1849) *Outlines of Astronomy*:

> "In celestial perspective, every point to which the view is for the moment directed, is equally entitled to be considered as the 'centre of the picture', every portion of the surface of the sphere being similarly related to the eye. Moreover, every straight line (supposed to be indefinitely prolonged) is projected into a semicircle of the sphere, that, namely, in which a plane passing through the line and the eye cuts its surface. And every system of parallel straight lines, in whatever direction, is projected into a system of semicircles of the sphere, meeting in two common apexes, or vanishing points, diametrically opposite to each other" (citation acquired from online version).

The implication of such perspective schemes is that lines that are straight in the world should not necessarily be perceived as straight by the viewer. The issue had been taken up experimentally by Helmholtz (1910/1962), who observed that the lines in a large-visual-angle grid seemed to curve inwards in the far periphery. This result is most easily checked by looking down at a tile floor in a large building (such as an airport). In the context of the eye moving around to explore the environment, a natural definition of straightness is the path followed by the eye as it moves between two points. Such motions are determined by the space-orientation of the axis around which the eye rotates, which is governed by Listing's law.

The question raised by Helmholtz was: what line geometry provides the ocular motion path for lines in the visual periphery, ie lines that do not pass through the fovea? Geometrically, the answer is *direction circles* (Klüster 1876; Helmholtz 1910/1962; Simonsz and Den Tonkelaar 1990), which have the property of being circles inscribed on the visual sphere that all converge to pass through the 'occipital point', or the point directly behind the viewer opposite the fixation point (figure 2). Direction circles are thus not great circles unless they pass through the fovea (along any meridian). Relative to these great circles, the other direction lines may be considered to be curved, but they will all remain self-congruent as the eye is rotated (as long as the rotation adheres to the minimum-energy principle of Listing's law). The direction circles thus correspond to a visual version of the *principle of conservation of energy*, first enunciated by Helmholtz (1847). They correspond to the spatial invariants, or null distortion field, for eye transformation under minimum-energy conditions.

As such, the direction circles form a kind of invariance 'horopter' for a given direction of eye-movement transformation, which in physics terminology would be termed the invariance domains of the minimum-energy rotational group. There is thus considerable interest in determining whether the visual perception of straightness conforms to this combination of minimum-energy principles. The reason that the spherical issue enters the question of natural perspective is that the eye cannot translate across a surface—the eye can only rotate about its center. A person can translate through the world (although only with some difficulty, unless he/she is on a moving vehicle). However, natural perspective is more typically associated with the stationary observer with a moving eye, which is capable only of rotation. It is this distinction that brings the direction lines into play as the candidate invariants for natural perspective.

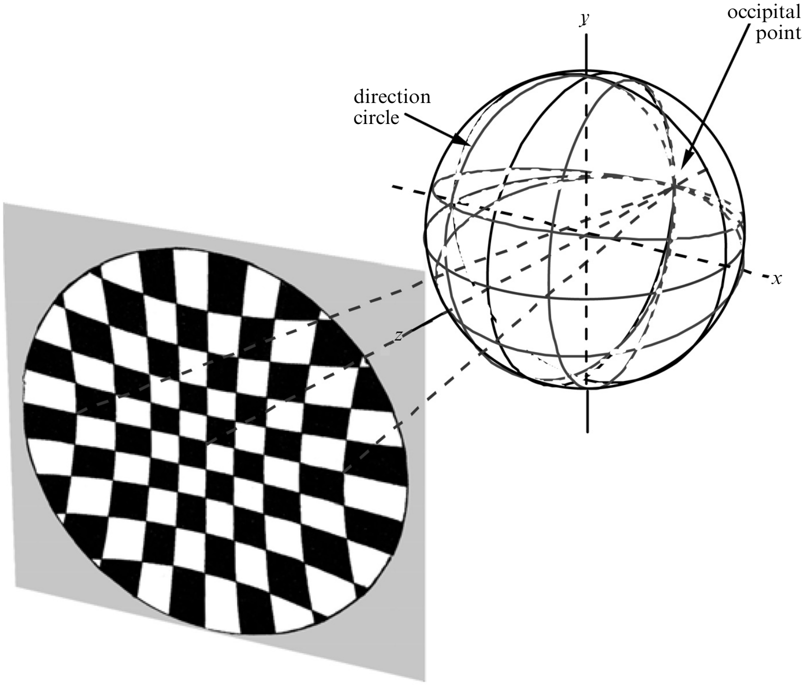

Figure 2. Ocular rotation circles, known as 'direction circles' and their projection to a frontoparallel plane. From Rogers and Rogers (2009).

Indeed, Panofsky (1924) mentions that the curved retina is a key reason for expecting straight lines to be curved in natural perspective. However, the preceding development has said nothing about the shape of the retina, and is framed purely in terms of the operational invariants of the optic array. On this view, it is not the curvature of the retina that is relevant, but the sphericity of the eye that determines the form of the invariants by constraining its motion to rotation around a point and adherence to Listing's law controlling the rotation axis for trajectories not passing through the primary position. In one sense, the two properties both derive from the curvature of the eyeball, but differ in the way its curvature influences the geometry of perspective.

This conflict between domains of operation may account for some of the discrepancies among the results of recent empirical studies of this issue. As reviewed by Rogers and Rogers (2009), for example, a wide range of results on the geometry of the perceived straightness of lines under 2-D and 3-D viewing, depend on the specifics of the viewing conditions. The above considerations suggest four factors that might interact to affect each other:

(i) the degree to which the tasks allow eye movements (enhancing the direction-line invariant) or require fixation;
(ii) the relative strength of binocular versus monocular cues to the flatness or curvature of the surrounding field;
(iii) the relative strength of the monocular and binocular cues to 3-D structure defining the edge to be judged;
(iv) the degree of realism and appeal to Bayesian priors for known straight edges of objects (in the translational sense).

Interestingly enough, both the study by Helmholtz and the rigorous replication by Rogers and Rogers (2009) find a highly accurate adherence to the minimum-energy prediction for the perceived visual straightness of peripheral lines under conditions of precise fixation, although this result is immediately violated as soon as the eye moves to fixate a peripheral line. This apparently paradoxical violation can be seen to make sense as soon as it is realized that the minimum-energy principle applies only to lines *parallel to the direction of the eye movement*. The eye movement required to fixate the peripheral line cannot be parallel to that line, and therefore does not bring the minimum-energy principle into play with respect to that line. Similarly, the checkerboard of figure 2 does not form a minimum-energy field because it has roughly orthogonal sets of features. Each set alone would form a minimum-energy field, but the two sets combined together violate the invariance property of the field for eye movements of any kind.

In conclusion, I have argued that straightness is most satisfactorily defined in terms of the minimum energy principle of eye rotation, both in a formal sense and as an operational definition available to an organism. For the physicist, translation is performed by hand or machine. Since the eye is not amenable to the operation of translation, its closest approximation is rotation about its center in the direction of the local line orientation. For such motions, the lines of eye movements away from the primary position form the direction circles identified by Helmholtz, which his and later studies appear to validate as the operative principle determining perceived straightness in human observers. These studies therefore appear to put the concept of perceptual straightness on a firm theoretical footing based on the kinetics of eye movements, and one that was anticipated by a wide variety of thinkers on the issue of curved perspective over the past millennia.

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