Global awareness in Da Vinci's youth

The contribution of Leonardo da Vinci¹ to many spheres of human endeavour are too well-known to need enumeration. However, one obscure aspect of his oeuvre is a complete world map, or mappamundi, to be found among his papers in the Royal Library, Windsor collection. With one exception, I have not seen this remarkable document mentioned in any of the large set of books on his works, including several authoritative surveys of his drawings and diagrams. And this despite extensive coverage of his cartographic output for the Renaissance rulers Ludovico Sforza in Milan, Piero Soderini in Florence, Cesare Borgia in the Marches around Arezzo, and Pope Leo X at the Vatican, almost all for a series of large-scale hydro-engineering projects that either never took place or could not be taken to completion due to unforeseen complications.

The sole exception to the lack of coverage of da Vinci’s global cartography is the extensive treatment in Veltman’s ‘Studies on Leonardo da Vinci’ (a book that is no longer obtainable), which details his scattered efforts to capture the geometry of the sphere.² Even here, however, the world map itself is mentioned only in passing, with no consideration of its place in the historical development of the cartographic representation of global geography.

One can ask when da Vinci first became interested in global geography. Was it in the early 1500s, after he left his long-established position in Milan as a result of what must have been a flood of societal interest in the world geography following Columbus’ discovery of the Americas? Or was it perhaps early in his life, as a result of early stirrings of interest in the nature of the globe when he first arrived in Florence? One item of interest in support of an early interest is the armillary sphere (or astrolabe) that he included in what is historically the first depiction of a perspective window for projecting accurate views of three-dimensional objects (Fig. 1). The drawing is generally dated to his first Florentine period, around 1480. The object chosen by da Vinci in this sketch is not a sculpture or a building, as might have been expected from his other output of this period, but a depiction of an analytic device of adjustable circles forming an armillary sphere, although it cannot be determined whether it is a celestial or a terrestrial sphere that is depicted. Whichever is the case, this choice attests to da Vinci’s interests in global matters beyond the Tuscan environs of Florence in which he grew up.

The second link to a larger scale conception is the presence of Paolo di Pozzo Toscanelli in the Florence of da Vinci’s youth. Toscanelli had been one of the progenitors of perspective geometry in discussions with Brunelleschi early in the century,³ then worked with Nicolas di Cusa to produce the classic Renaissance text on squaring the circle⁴ and other philosophical matters before turning his attention to cartography. As a cartographer, he was in demand well beyond the shores of Italy, providing maps of the Mediterranean to rulers such as King John of Portugal in 1460. Moreover, as an astronomer, Toscanelli detailed his observations on what later became known as Halley’s comet, three centuries before Edmund Halley’s notation of it. Importantly, as an astronomer, Toscanelli detailed his observations on what later became known as Halley’s comet, three centuries before Edmund Halley’s notation of it. Importantly, as an astronomer, Toscanelli detailed his observations on what later became known as Halley’s comet, three centuries before Edmund Halley’s notation of it. Importantly, as an astronomer, Toscanelli detailed his observations on what later became known as Halley’s comet, three centuries before Edmund Halley’s notation of it. Importantly, as an astronomer, Toscanelli detailed his observations on what later became known as Halley’s comet, three centuries before Edmund Halley’s notation of it. Importantly, as an astronomer, Toscanelli detailed his observations on what later became known as Halley’s comet, three centuries before Edmund Halley’s notation of it. Importantly, as an astronomer, Toscanelli detailed his observations on what later became known as Halley’s comet, three centuries before Edmund Halley’s notation of it.

Fig. 1  Drawing of a man using a perspectograph to draw an armillary sphere. Leonardo da Vinci (Codex Atlanticus I, 5r; ~1480). Courtesy of Pablo Garcia.

The unveiling of this remarkable intersection of science and religion took place to great public fanfare.
in 1468. As chief pupil of the workshop of Andrea Verrocchio, the heir to Donatello and Michelozzo as the principal artisanal provider to the ruling Medici clan, da Vinci would certainly have been present as one of the supporting cast at this momentous event, and may even have performed musically to celebrate it (as he was famous in his youth for composing extempore songs at public events).

Da Vinci himself had a close connection with the Duomo at that time, when he was an impressionable young man in his late teens. In the same year, 1468, his master Verrocchio received a commission from the Medici to instal a gilded copper sphere on top of the lantern surmounting the Duomo (Fig. 2). The sphere again represented a kind of global consciousness and weighed over 2 tonnes, requiring extensive use of hoisting machines that were still available from when Brunelleschi was erecting the dome itself. As Verrocchio’s pupil at the time, da Vinci would have been intimately involved in every phase of the operation, particularly as Verrocchio was renowned for being highly collaborative with his team of pupils. Thus, he would have had the opportunity to inspect the aperture of Toscanelli’s gnomon for himself while on the top of the dome. He would also have been vividly aware of the god-like view of Florence and the surrounding countryside from the 115 m height of this crow’s nest location, which may have played a role in his lifelong obsession with cartography and with flight, both in terms of the analysis of the flight of birds and of the design of a human flying machine.

A further link of da Vinci to global cartography is found in a fresco by his friend Donato Bramante from his early years in Milan (Fig. 3). The painting depicts the Greek philosophers Heraclitus and Democritus, representing pessimism and optimism, respectively, flanking a geographically detailed depiction of the globe. The two figures are generally accepted as depictions of da Vinci and Bramante himself, respectively. Their association with the globe seems to reflect a mutual interest in the large-scale conception of the land they inhabited, consistent with the idea that da Vinci brought the global interests that he had developed in Florence with him to Milan when he relocated there in 1482.

The final association comes in a sketch by Michelangelo of a philosopher holding a globe (See top image of Contents page) that is often considered to be a portrait of the aging Leonardo da Vinci (especially as it seems to be the model for Vasari’s portrait of him in his  Vitae). The fact that the

Fig. 2 The 22 m high lantern was completed by Michelozzo in 1461, and the 8 tonne gilded copper ball was added by Verrocchio and Leonardo da Vinci in 1469.

Fig. 3 ‘Crying Heraclitus and Laughing Democritus’ by Bramante, 1483.
philosopher is holding a globe has nevertheless been considered puzzling in relation to this attribution.

**Da Vinci’s World map**

Despite Leonardo da Vinci’s renown as a Renaissance man, and as a cartographer in particular, he is not generally acknowledged as authoring a world map (or mappamundi) of the geography of the world. Nevertheless, there is a world map among his papers in the Royal Library, Windsor (Fig. 4), which is one of the very first maps to name the Americas, and has the correct overall configuration of the continents, including an ocean at the north pole and a continent at the south pole (at the centres of the left and right quartets, respectively). At that time (as subsequently), there was a wide variety of projective conventions attempting to depict the curved surface of the globe onto the planar surface of a sheet of paper. By breaking the surface into eight octant petals in two florets, da Vinci develops a unique projection that had by far the most isometric mapping geometry to that date (although at the cost of a set of crosscuts that split some of the continents).

Da Vinci’s projection may be compared with a modern mapping to the same projection (Fig. 5). This mapping makes it clear that, although he has the right general idea, he has substantially exaggerated the size of Europe and has the Americas much too far to the west (somewhere near Hawaii). Nevertheless, he shows India, Malaysia, China, Japan and even the Russian Far East peninsula in approximately the right proportions, implying that he must have had access to some information from sailors of the oriental sea routes. Remarkably, also, he has approximately the correct dimensions for both the Arctic Ocean and the landmass of Antarctica (at the centres of the right and left florets, respectively), something achieved by no other cartographer of the era, or for the next two centuries! The Arctic is particularly surprising in view of the fact that it is continuously covered in a thick layer of ice.
Attribution to Leonardo da Vinci

Despite the fact that it was found in his collection of papers from those assembled by his pupil Francesco Melzi, the attribution of this mappamundi to Leonardo da Vinci has long been questioned by most da Vinci scholars, and has consequently been omitted from almost all books in his oeuvre (the sole exception being a passing mention in Veltman). The line of the coastline is considered too curly and the notations too plain to match da Vinci's other works; thus, although it is occasionally acknowledged as his overall conception, the execution is commonly considered to be the work of a pupil. It should be noted, however, that all the features discussed to this point are those of conception, and thus should be accepted as relating to his autograph contributions to the design of the map, even if it is a copy of his original.

More recently, key evidence of da Vinci’s ownership of the world map has emerged from references to it in the Codex Atlanticus. He was a good friend of the rich merchant’s son, Giovanni di Amerigo di Benci, brother of the 17-year-old Ginevra di Benci whose portrait da Vinci had painted in 1473 (now in the National Gallery, Washington DC). The two young men evidently had a close working relationship, because da Vinci has notes among his papers in the Codex Atlanticus about exchanging books, precious stones and supplies (such as ‘brass for eyeglasses’) with Giovanni Benci. It was to Benci that da Vinci entrusted his most ambitious work of that period, the full-scale cartoon for the unfinished ‘Adoration of the Magi’, when he left Florence for Milan in 1482. Of particular relevance here is the scribbled reference in his notes to ‘my world map that is with Giovanni Benci’, which is unequivocal evidence of an interest in global cartography at that time, and that his interests extended beyond the shores of Italy to the configuration of the lands of the entirety of the globe. There is a suggestion of consequentiality about the statement, as though it was more than a sketch on a folio sheet. One has the image of a notable document that was worth recording as having been loaned to a particular friend.

What remains ambiguous, however, is whether the ‘my’ refers solely to ownership or implies that it was a map that he had drawn himself. Was da Vinci compiling world maps in his first Florentine period, before the age of 30? And did this indeed tie in with Toscanelli’s cartography expertise of the previous decade? Or was it a world map that he happened to own somehow (as a gift from Toscanelli, for example)? It seems likely that the statement would have identified the source of the map in this case, as in ‘Ser Paolo’s world map that I left with Giovanni Benci’. Seen in this light, the statement does seem to imply that da Vinci was compiling a world map in this early period. Presumably this cannot have been have been the map in Figure 4 because that includes the American continent, whose existence was not suspected until after 1492, but an earlier attempt at a world map that has not survived. An alternative interpretation is that the comments about exchanges with Benci were from da Vinci’s later post-1500 period in Florence, and that the map was the one we now have, although evidence of exchange between the two during this later period is much less secure.

With respect to his actual authorship, it is interesting to note that da Vinci’s cartographic style is substantially different from his loose and flowing drawing style. His cartographic style can be seen in his map of the western Mediterranean (Fig. 6), which is also one of the few of his maps known beyond the borders of Italy, helping to underline the global reach of his cartographic interests. This accepted autograph map helps to link the mappamundi to da Vinci in two ways. The line style can be seen to have the same curly character and the lettering to have a similar plain character to those in the mappamundi, although the letters are predominantly lowercase as in most of his maps. The switch to upper case can be hypothesised to be indicative of an intention to provide the mappamundi to a sponsor of some kind, such as Cesare Borgia or Giuliano de Medici, both of who were appointed to French Dukedoms (Borgia as Duc de Valentinois, Medici as Duc de Nemours), the French honorific titles reflecting the international range of their respective ambitions.

Fig. 6 Map of the eastern Mediterranean and drawing of the tower of the Palazzo della Signoria (Codex Atlanticus, 1106r, 1496). Courtesy of Biblioteca Ambrosiana.
A third key link to da Vinci’s works is a remarkable, but again little known, page from his notebooks containing an intensive exploration of what seem to be all the known cartographic projections of the globe at that time (reproduced in Fig. 7). Each miniature sketch represents a different projection geometry, with several of them showing projection lines or shadows, clearly indicating that he understood them as geometric projections of the globe, not simply images from some maps that he may have had to hand. The particular relevance of this page for the present analysis is that, among this array of classic projection geometries, da Vinci includes (at lower right) a clear reference to the octant triangular projection, tightly coupling this evidently autograph sheet to the conception underlying the mappamundi. Taken together with Figure 6, this notebook page seems to remove any grounds for questioning da Vinci’s authorship of the unique mappamundi. Even if it is in fact a copy by a pupil, the pupil is most unlikely to have the kind of knowledge required to make major modifications, so we are safe in assuming that all the characteristic details are reflective of da Vinci’s own intentions.

To highlight the variety of projections considered in this notebook page, they have been isolated in Figure 8 and annotated with the originator of each form of projection (as far as they are known). Three of them date back to Roman times, as represented by the Geographia of Ptolemy, which had recently (in 1410) been translated and distributed by Jacopo da Scarperia, and of which da Vinci was known to have owned a printed copy with illustrations. He must have been among the many whose global conceptualisations were expanded and regularised by this systematic presentation of geographic information. Few, however, are likely to have gone into such depth in the analysis of the forms of projective geometry, and the scope of da Vinci’s analysis is a further reason for assuming that he most likely sought out the counsel of an experienced cartographer like Toscanelli for discussions of these ideas.

The Reuleaux triangle projection
It is noteworthy that da Vinci’s world map has a unique cartographic projection onto eight spherical-geometry triangles that provide close to isometric projection throughout the globe.

Fig. 7 A page of Leonardo da Vinci’s cartographic notes (Codex Atlanticus, 521r, 1478-1519). Courtesy of Biblioteca Ambrosiana.
Fig. 8  Identification of the precursors or earliest known examples of nine different projections explored by Leonardo da Vinci in his notebook page. For those projections dated later than 1508, his drawings should be effectively considered the original precursors.

Fig. 9  Details of pages from Leonardo da Vinci’s notebook developing the geometry of his octant triangle figure, which was later developed as the Reuleaux triangle (Leonardo da Vinci, Paris Manuscript A, 15v; Codex Atlanticus, 923r).
These are triangles created from three arcs of circles with their centres at the apices of the triangle, and as such, were easily constructible with the compasses available in Renaissance times (Fig. 9). The spherical triangle in this form is known as the Reuleaux triangle, after Franz Reuleaux (1829–1905), who realised that it would roll with a constant diameter. There is no evidence that da Vinci was aware of this property of its dynamics, despite his geometric construction of it from the three vertices of a Euclidean triangle with a compass set as a constant angle of its points. It seems appropriate, therefore, to say that da Vinci had anticipated the Reuleaux triangle rather than he invented it in this rolling sense, per se. Nevertheless, he was clearly fascinated with the geometric properties of this non-Euclidean form of a triangle and was certainly the first to develop it into a quasi-isometric mapping projection.

Da Vinci was evidently aware of the geometric significance of this figure, as he develops it as a means of constructing a regular pentagon (Fig. 9, right). If connected at one apex, equilateral triangles would form a hexagonal figure. To employ them to construct a pentagonal shape, da Vinci has subdivided the height into five subdivisions and indicated that the topmost division should be the centre of combination. This is a very good approximation to a valid construction, because the angle at the centre derived in this manner is 71.6° instead of the required 72°, which is accurate to 99.44%. Whether this pentagonal construction relates to other geometric figures that he developed, such as the Vitruvian Man, is unclear.

**Geometric development**

Da Vinci is renowned for the range of geometric explorations in his notebooks. One such development that relates to his octant approach to world mapping is that of ‘squaring the circle’, which has already been noted as a key interest of Toscanelli. Da Vinci’s interest is often referred to his iconic illustration of the geometric relationships in human proportions developed by the Roman architect, Vitruvius, known as the Vitruvian Man. Since, however, those proportions and the configuration of the elements were indeed worked out by Vitruvius, it is not clear from this what level of understanding da Vinci needed to have had in order to construct this diagram. In this connection, therefore, it is of interest to find explicit analyses of the relationships between the circle and its circumscribing square in his notebook pages, such as that reproduced in Figure 10.

At lower left, he develops an octagonal analysis of the area of a circle inscribed in a square rhombus (regular diamond shape) by approximating it with the eight triangles making up an inscribed octagon, reconfigured as an ‘accordion’ figure of eight isosceles triangles (or, actually, seven isosceles triangles and two right triangles making up the eighth, labeled $abcdefgh$) together with a lenticular segment labeled $K$ to account for the eight missing lenticular elements in the octagonal approximation to the circle. This large lenticular segment $K$ should have an area eight times larger than that of each of the eight segments in the circular figure, giving it a linear extent of $\sqrt{8}$, or $2\sqrt{2}$. Deriving this ratio seems to be the motivation for the scaled figures of the semicircle, quadrant, octant and sector, which all fill the same area relative to the original circle, making it clear that the lenticular segment is in the correct ratio of $\sqrt{8}/4 = 0.707$ to the width of the eight triangle figure $abcdefgh$. (Fig. 10).

**Fig. 10** Octagonal analysis of the area of a circle inscribed in a square rhombus (left-right reversed) in which da Vinci is developing the idea that the circle, semicircle, quadrant, octant and sector all fill the same area relative to the enclosing rhombal square. (Leonardo da Vinci, Royal Library, Windsor, 12700v; detail).

This analysis of the area of the circle does not solve the problem of squaring the circle, because there is apparently no indication how to derive the area of the lenticular segment, but it does show da Vinci thinking about the progressive fractional subdivisions of circles in a way that can be readily generalised to the solid geometry of spheres. It also links da Vinci directly with Toscanelli in the analysis of the problem of squaring the circle, which at one point he thought he had actually achieved: ‘On the night of St Andrew I
reached the end [goal] of squaring the circle and at the end of candlelight, of the night and of the paper on which I was writing, it was completed’. (Madrid Manuscript II, 112a).

A further analysis in which he took a similar approach to the spherical geometry per se is found in the Paris Manuscript (Fig. 11). Here da Vinci subdivides a sphere into octants and explores different ways of segmenting (or digitising!) the volume of an octant to quantify its volume. Indeed, this analysis may be considered a precursor to the differential calculus developed formally by Newton and Leibniz nearly two centuries later. Moreover, some of the approaches on these pages involve explicit visualisation of the spherical octant as a Reuleaux-like spherical triangle, which may have been what suggested the Reuleaux triangle approach to the world map.

Role in the discovery of America
This paper began with the evidence that da Vinci had a global conceptualisation of the world in his youth, and would most likely have encountered Toscanelli in his role in the leading workshop in Florence working at the Duomo at the same time as Toscanelli’s gnomon. Now, as a cartographer, Toscanelli was in communication with the authorities in Portugal and Spain, and was reputed to have developed the (erroneous) map that inspired Columbus to set out to discover the western route to the Indies (which were known from the overland route through Asia). Toscanelli lived to 1482 (about the time that da Vinci left Florence), so his map must have been drawn at least a decade before Columbus set sail, and in the period when he would have known da Vinci. This gives rise to the speculation that da Vinci may have been involved in the conversation about the extension of the Ptolemaic
Evidence that da Vinci had a larger conceptualisation of global exploration is found in an allegorical drawing of unknown date, but undoubted provenance (Fig. 12). (Notice the characteristic Vinciesque rocks in the background at left.) This drawing shows a seafaring animal (apparently a wolf) using a compass-like device to target an eagle perched on a globe. The emblem of Isabella, the Queen of Spain in that era, has been inset for comparison, supporting the suggestion that the navigation target represents the Spanish throne (which was the actual patron of Columbus’ voyages to the Indies). The suggested interpretation is that this image represents the conceptualisation of global exploration under the Spanish aegis, and that da Vinci was indeed somehow involved in that enterprise. Da Vinci was also an acquaintance of Amerigo Vespucci, whose voyages to the New World in the 1490s established the continents of the eponymous Americas, and according to Vasari had a sketched portrait of him among his notebook papers. Vespucci was a cousin of the Medici and also of Simonetta Vespucci, whose wedding to her cousin Marco Vespucci at the Medici Palace in that same year (1469) was the inspiration for Botticelli’s famous paintings of the ‘Birth of Venus’ and ‘Primavera’.

Later influences
Although barely recognised in the canon of cartographic projections, da Vinci’s octant projection can be seen to have influenced a line of cartographers to the present day. The first example of this influence was the diagrams of Oronce Fine (1494–1555), a French cartographer active in the years just after da Vinci died in France, who was notable for ‘crossing swords’ on academic matters with King François I, and who developed the gridlines of the spherical octant projection and elaborated it into several formats, including a whimsical heart-shaped projection (Fig. 13). There was a further flurry of interest in octant maps the following century when Daniel Angelocrator (Engelhardt) produced several rigorous versions.14 (Unlike da Vinci, all these successors had Antarctica dramatically too large, however.)

We may jump forward several centuries to the Pan-Pacific Exposition of 1915 announcing the recovery of San Francisco from the great earthquake of 1906,
for which Bernard J. Cahill developed a connected version of the da Vinci octant projection to map the challenge of a flight around the world (Fig. 14). How da Vinci would have appreciated this combination of two of his lifelong interests! To provide more extended connection regions, Cahill slightly distorted the da Vinci octants into figures with each edge consisting of three straight segments, then connected the four pairs of north-south quasi-octants into a butterfly-shaped figure that allowed all the continents except Asia and Antarctica to appear as connected landmasses. Many of the continents are rotated to extreme angles relative to their natural north-south axes, however.

Following Cahill, several cartographers developed further forms of segmented maps, by projection onto various regular polyhedra, notably the Buckminster Fuller octahedron (and the other regular polyhedra) and the Keyes projection that took a different configuration of the Cahill projection to avoid the cut through central Asia (but still lopped off the Russian Far East peninsula and segregated New Zealand from Australia, however). The Keyes projection also had continents such as Africa tilted at a rather severe angle.

These deficiencies may be partially rectified with a Cahill-like projection that does not adhere to the principle of complete octants, but allows segments of the octants to be shifted to complete the continents and also to regularize the oceans somewhat. A final map taking this approach, developed by the author, is depicted in Figure 15. Part of the motivation for including this reconfigured projection is to illustrate how powerful the da Vinci approach was in providing
for closer-to-isotropic projection of the relative shapes and sizes of all countries and continents throughout the globe. All other projections distort the shapes of at least one continent, and most projections vastly exaggerate the sizes of Greenland and Antarctica. Here one can clearly see the approximate equivalence in size of the United States, Brazil, China, Australia and Antarctica, for example. Distances, too, are well represented by straight lines on the map within continents. The main sacrifice is of distances across oceans, which are heavily distorted in this projection. It may be possible, however, to derive an equivalent projection preserving the shapes and size of the oceans in the same way, although it is likely to be more challenging because they cover the majority of the earth’s surface.

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Notes
1 A note on naming conventions. As opposed to the art-historical consensus, I will follow the convention of referring to him by his last name, in common with those of other intellects of the time, such as Columbus, Toscaneli, Bramante and so on. It could be argued that his contemporaries would have referred to him by his first name, but it is well documented that many used his full name, and it seems over familiar of present-day authors to associate themselves with the appellations of his contemporaries.
4 Nicolo di Cusa (with Toscaneli), Dialogus de ponderibus, monentis, et mensuris per totum terrarum orbem usitatis, Frankfurt: Johann Berner, 1628.
5 ‘In the Codice Atlantico we find sketches of a globe being unfolded that is not far from Waldseemüller’s pseudo-Ptolemaic, cordiform projection (1507)’, Veltman, 1986.
9 Veltman, 1986.
13 Di Cusa, 1457.
14 D. Angelocrator, Doctrina de ponderibus, monentis, et mensuris per totum terrarum orbem usitatis, Frankurt: Johann Berner, 1628.

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